ETH
Eidgenössische Technische Hochschule Zürich
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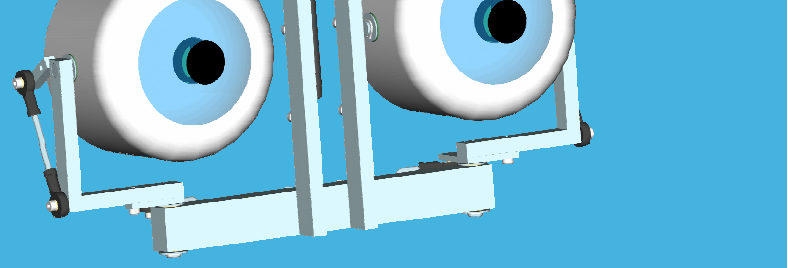
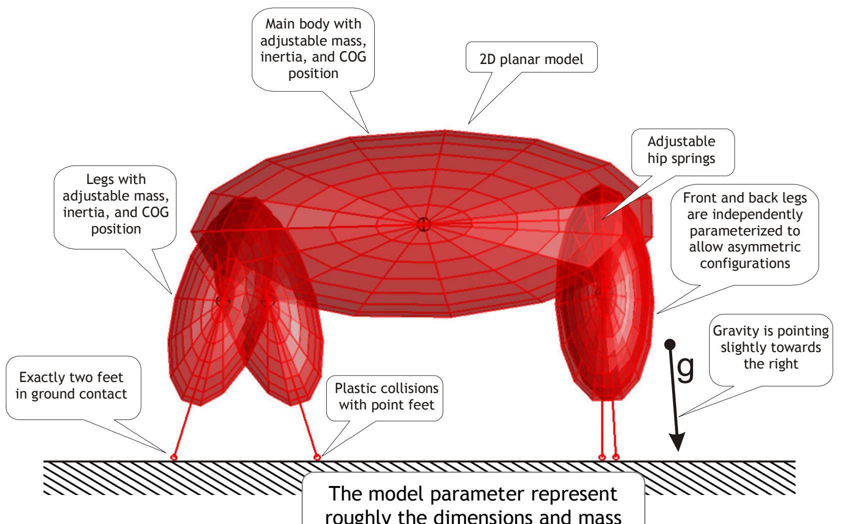
AUTONOMOUS SYSTEMS LAB



Passive Dynamic Walking with Quadrupeds

The Model



The model parameter represent roughly the dimensions and mass properties of a Merino Sheep

Model Parameters	
Main Body Mass	80 Kg
Main Body Length	1.2 m
Main Body COG Position	Center
Leg Mass	5 Kg
Leg Length	0.8 m
Leg COG Position	0.25 m below hips
Ground Inclination	1 deg
Hip Spring Stiffness	0 Nm/rad

- The Models state is fully defined through the following 7 variables:
  - o Angles of the back stance leg and the two swing legs
  - o Angular velocities of these legs
  - o Distance between front stance and back stance foot

Contact Modeling

The mass matrix M and a matrix W of partial derivatives of the gap function g at impact define the symmetric matrix $G = W^T M^{-1} W$. Its elements describe the contact coupling[1]. Off-diagonal elements in G indicate that different contacts are coupled. → In such a case the order of treating contacts becomes substantial to the outcome of the simulation.

- This phenomena can be observed in a passive dynamic 2-beat gait
- It introduces asymmetries which prevent a true 2-beat gait
- The (numerical) decision of contact order as a great impact on the post-collision velocities (see Table)

	Back First	Front First	Simultaneous
ω Back Stance	-0.939	-0.939	-0.912
ω Back Swing	-0.658	-0.693	-0.607
ω Front Swing	-0.693	-0.658	-0.607
Energy dissipation	14.8 %	14.8 %	19.5 %

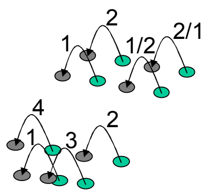
[1] C. Glocker, 2001 *On frictionless impact models in rigid-body systems*. Phil. Trans. R.Soc. Lond. A (2001)

PDW Gaits

There are three possible Passive Dynamic Walking gaits:

'Pace'[1]
'Trot' } Two Beat

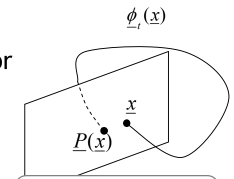
'Amble' Four Beat



[1] Note: Usually 'pace', 'trot', and 'amble' refer to running gaits, which might be misleading as there is no air phase and no elastic energy storage in passive dynamic walking.

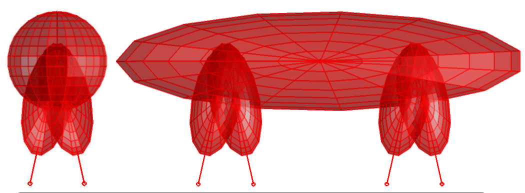
Methods

- Limit Cycle Identification:
  - o Numerical root search for $x^{k+1} = P(x^k)$
- Stability Analysis:
  - o Numerical linearization of $\Delta x^{k+1} \approx J \cdot \Delta x^k$
  - o Eigenvalue analysis of J
- Implementation in MATLAB



The Poincaré map is defined at back ground contact

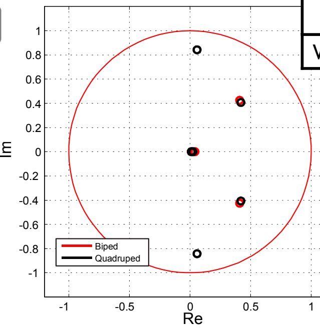
Comparison Quadruped - Biped



Models to study the impact of kinematic coupling

- Focus on kinematic connection:
  - o Radius of gyration of the main body equals the distance COG - Hip joints.
  - o No dynamic coupling
  - o The quadruped should behave like two independent bipeds.
- The additional Eigenvalues are stable.
- Small variations in velocity and Eigen-values are due to the coupling of contacts.

Gait		Biped	Quadruped
Eigenvalues		0.410 + 0.425i	0.420 + 0.407i
		0.410 - 0.425i	0.420 - 0.407i
		0.042	0.024
		-	0.059 + 0.843i
		-	0.059 - 0.843i
		-	0.011
Velocity		0.5131 m/s	0.5165 m/s

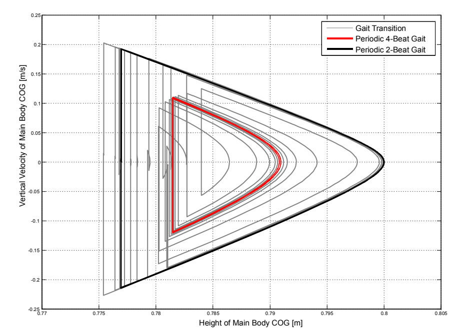


Comparison of biped and quadruped PDW / root locus plot

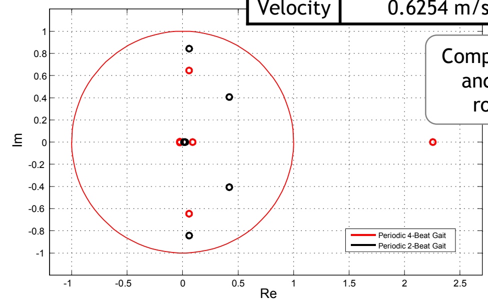
Four-Beat & Two-Beat Gait

- The four beat gait consumes less energy due to slower impacts.
- The 'phase-mode'-Eigenvalue is unstable and will lead to a transition into a 2-beat gait

Gait		4-Beat Gait	2-Beat Gait
Eigenvalues		-0.027 + 0.004i	0.062 + 0.846i
		-0.027 - 0.004i	0.062 - 0.846i
		0.130 + 0.616i	0.455 + 0.399i
		0.130 - 0.626i	0.455 - 0.399i
		0.081	0.011
		2.372	0.023
Velocity		0.6254 m/s	0.56434 m/s



Phase portrait: Transition 4-beat to 2-beat gait.

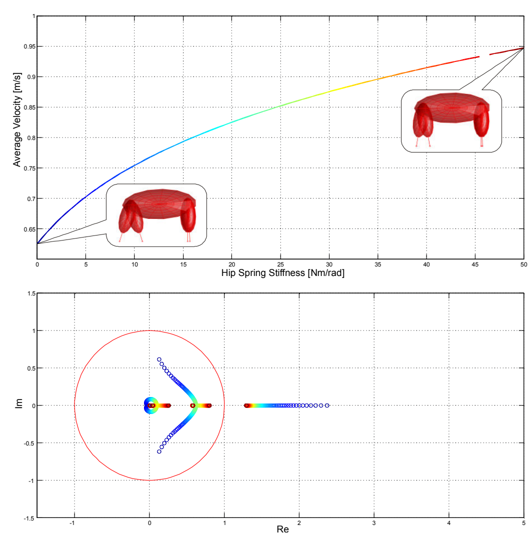


Comparison of 4-beat and 2-beat gait / root locus plot

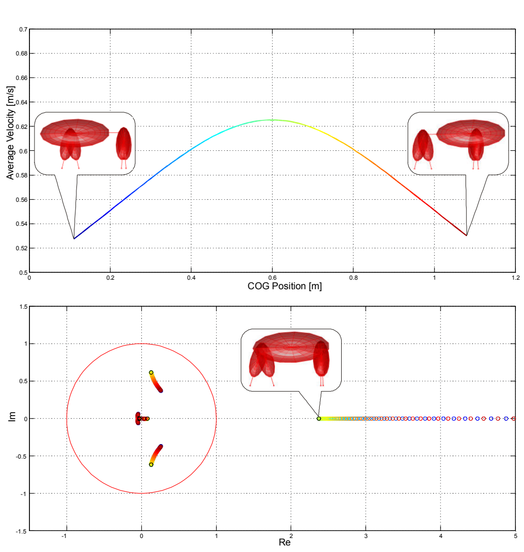
Parameter Variation & Stability of the Four-Beat Gait

- Limit cycle identification and stability analysis were performed for varying sets of model parameters
- For none of the variations studied a stable 4-beat gait was found
- This is consistent with Smith and Berkemeiers[1] claim that '... this ['phase'] mode is almost invariably unstable...'
- While the influence of hip springs is beneficial for the stability of this mode, any asymmetry between back and front legs will degrade it

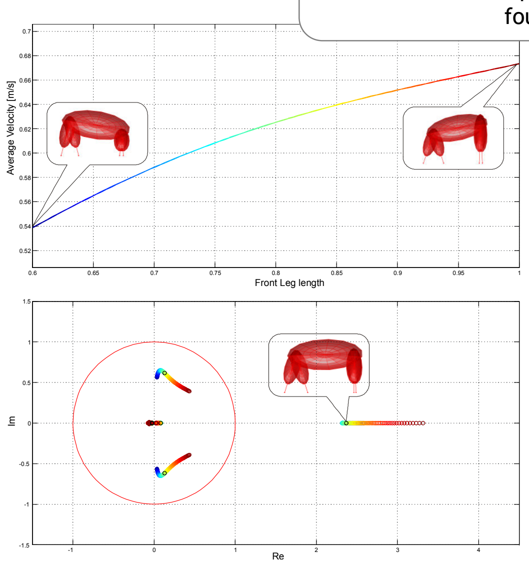
The graphs below show the walking speed of a periodic solution together with the corresponding root loci as a function of a selected model parameter. Missing data indicates that no periodic solution was found.



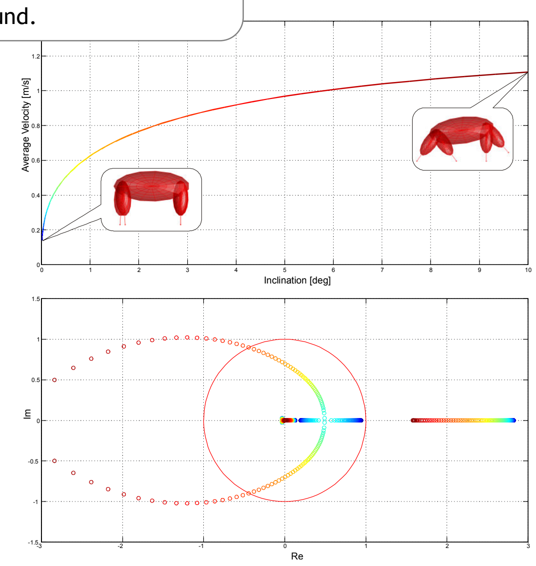
Influence of hip spring stiffness



Influence of main body COG-shift



Influence of leg length asymmetry



Influence of inclination

[1] A. Smith and M. Berkemeier, 1997 *Passive Dynamic Quadrupedal Walking*. Proc. of the Int. Conf. on Robotics and Automation

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