FOOT PLACEMENT AND BALANCE IN 3D

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INTRODUCTION

Dynamic walking involves instability and balance restoration. Humans are unstable for at least 80% of the gait cycle and use foot placement to restore balance [1]. Wight et al. [2] derived the foot placement estimator (FPE) to find a foot placement location that will take a planar, simplified biped from an unstable state to a statically stable state. The FPE has been used to dynamically balance a bipedal robot [2] and to analyze human foot placement and balance in the sagittal plane [3]. This work presents the latest efforts to extend Wight et al.'s work to 3D (3DFPE).

MODEL

A 3D inverted pendulum with a circular contact area is a simple 3D model equivalent to Wight et al.'s 2D planar biped (Fig. 1). The model consists of a body with mass and inertia on top of a massless contact ring that has rolling resistance. The system equations are very similar to those of an Euler disc [4], and have been omitted for brevity. The model is Lyapunov stable and was been used to calculate the regions of stability and validity for the 3DFPE (Fig. 1).

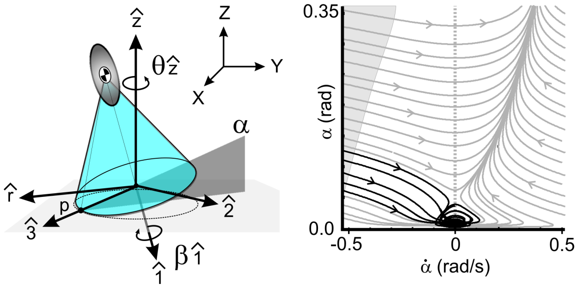
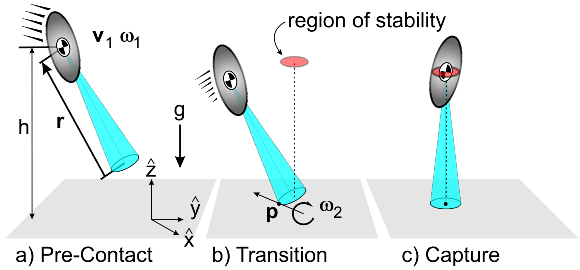


Figure 1: Three angles (α, β, and θ) and angular velocities are used to model the motion of the pendulum. A phase portrait over α and $\dot{\alpha}$ ($\dot{\theta} = 1$, $\dot{\beta} = 0$) is shown for a pendulum that approximates human proportions. Stable trajectories are in black; invalid regions are shaded grey.

The 3DFPE is the foot placement location that will cause the inverted pendulum to transition to a statically stable state (Fig.2). This location can be found using impulse-momentum and energy conservation laws if six assumptions are made: the foot sticks and does not slip; the point of contact (p) does not move; the mass (m), inertia ($\mathbf{J}$) and leg length ($|\mathbf{r}|$) stay constant after contact; and the pendulum rotates in a vertical plane that intersects the region of stability. The inverted pendulum will transition to a statically stable standing pose if it rotates towards the stable region after contact and has enough kinetic energy to enter the stable region but not enough to exit.

The angular velocity of the pendulum after contact can be found using impulse-momentum (Eqn. 1).

$$\mathbf{h}_p = \mathbf{J}\boldsymbol{\omega}_1 + \mathbf{r} \times m\mathbf{v}_1 = \mathbf{J}\boldsymbol{\omega}_2 + \mathbf{r} \times m(\boldsymbol{\omega}_2 \times \mathbf{r}) \qquad (1)$$



a) Pre-Contact b) Transition c) Capture

Figure 2: Stepping on the 3DFPE will cause the unstable pendulum (a) to transition (b) to a statically stable state (c).

Foot contact locations that cause the pendulum to rotate in a vertical plane towards the region of stability satisfy Eqn. 2.

$$\boldsymbol{\omega}_2 \cdot \mathbf{r} = 0, \ \ \boldsymbol{\omega}_2 \cdot \mathbf{g} = 0 \qquad (2)$$

The pendulum must have just enough energy to enter the stable region but not enough to exit. This is only possible if the system is able to lose energy either through rocking or rolling resistance once it enters the stable region. The pendulum will enter the stable region if the constraints in Eqn. 2 are met and the system energy is slightly greater (by δ) than the peak potential energy after losses (ϵ_L) are taken into account. In the case of human gait the energy losses should be insignificant [3].

$$\frac{1}{2}m(\boldsymbol{\omega}_2 \times \mathbf{r})^2 + \frac{1}{2}\boldsymbol{\omega}_2 \cdot \mathbf{J}\boldsymbol{\omega}_2 + mgh - \epsilon_L - \delta = mg|\mathbf{r}| \quad (3)$$

SIGNIFICANCE

The 3DFPE is highly useful for developing 3D balance controllers and measuring the balance performance of both humans and robots. The distance between the biped's COM and the 3DFPE is a measure of instability. The distance between the biped's foot placement and the 3DFPE is a measure of balance performance. The 3DFPE will make it possible to fairly compare bipedal instability and balance performance between bipeds of unequal size in a theoretically grounded manner. Experiments that compare the 3DFPE to human foot placement are in progress.

REFERENCES

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