

Non-minimum phase problem of ZMP stabilization

¹Houman Dallali, ¹Martin Brown, and ²Bram Vanderborght

¹The University of Manchester, Control Systems Centre

²Italian Institute of Technology; email: houman.dallali@postgrad.manchester.ac.uk

SUMMARY

In this abstract, the non-minimum phase problem of the cart table model is demonstrated and a new model is proposed that does not have this characteristic and can potentially overcome the problem of undesired undershoot and limited bandwidth. The new pendulum-cart-table model utilizes the torso motion to achieve a desired bandwidth for the ZMP feedback stabilizer that is used for dynamic balance of position controlled humanoid robots.[1]

INTRODUCTION

Zero Moment Point (ZMP) is one of most widely used dynamic stability criterion for fully actuated bipeds and especially humanoid robots. ZMP can be used to generate stable walking trajectories but these trajectories can not realize stable walking in practice due to imperfect ground conditions, modelling errors and unknown disturbances. Therefore a real time feedback stabilizer must be used to adapt the trajectories based on the sensor information. The cart table model [2] is a convenient method to produce the walking trajectories, but it shows inherent limitations for

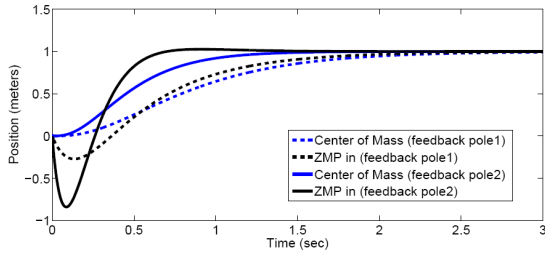


Figure 1: Step response of the closed loop cart table system with poles at pole1= [-2,-4,-6], pole2= [-4,-6,-10]. Note the amount of undershoot associated with pole2 which has a higher bandwidth.

feedback stabilization due to its non-minimum phase characteristics. This is illustrated as a step response of this model in figure 1. As it is shown in figure 1, there is a trade off between the amount of bandwidth required for disturbance rejection and the amount of undershoot that must stay within the stability margins.

METHODS

The non-minimum phase characteristic is an inherent property of the cart table model. In order to address this problem, the pendulum cart table model is proposed to add more degrees of freedom to the ZMP feedback stabilization problem by having two control inputs that can affect the hip (cart) and torso (pendulum) accelerations independently. The model is shown in figure 2. The computed ZMP for this model is given by eq.(2).

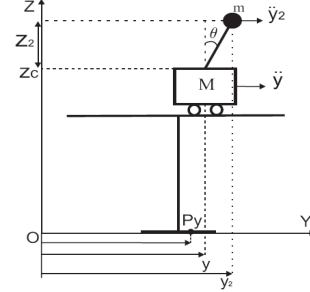


Figure 2: Pendulum cart table model.

$$\tau_{ZMP} = Mg(y - P_y) - M \ddot{y} z_c + (mg - m\ddot{z}_2)(y_2 - P_y) - m\ddot{y}_2(z_c + z_2) = 0 \quad \text{eq.(1)}$$

And from figure 2 we can write:

$$z_2 = l \cos(\theta), y_2 = y + l \sin(\theta)$$

Where, l is length of the pendulum. After simplification we get the following relationship:

$$P_y = y + a_1\theta - a_2\ddot{y} - a_3\ddot{\theta} \quad \text{eq.(2)}$$

$$a_1 = \frac{mgl}{(M+m)}, a_2 = \frac{Mz_c + m(z_c + l)}{(M+m)g}, a_3 = \frac{m(z_c + l)}{(M+m)g}$$

The transfer function of this system can be obtained as:

$$G(s) = \left[\frac{1 + a_1s^2}{s^3} \quad \frac{a_2 - a_3s^2}{s^3} \right] \quad \text{eq.(3)}$$

Since the transfer function of eq.(3) does not have any transmission zeros, one can use the new model to avoid the undershoot which is associated with the non-minimum phase property of the old model.

RESULTS AND DISCUSSION

To conclude, the limitations of cart table model for the purpose of ZMP feedback stabilization is shown and a new model is proposed. Feedback stabilization has to be real-time and therefore it has to have a high bandwidth to react to disturbances measured by the robot's foot sensors. This is an important part of control software architecture that adds robustness to the system. From trajectory generation point of view both models can use the preview control method [2], while the new model can also produce trajectory of the torso which can be used to move the upper body when the robot is climbing hills.

REFERENCES

1. Kajita, S., Espiau, B., Springer Handbook of Robotics. Chap. 16. Sec.4, Legged Robots. 2008.
2. Kajita, S., Kanehiro, F., Kaneko, K., Fujiwara, K., Harada, K., Yokoi, K., Hirukawa, H.: Biped walking pattern generation by using preview control of zero-moment point. , ICRA, vol.2, pp.1620-1626, 2003.