Dynamics and Stability of Lateral Plane Locomotion on Inclines

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SUMMARY

An actuated, bipedal model is developed to examine lateral plane locomotion dynamics and stability on inclines. Variations in the force-free leg length are determined via inverse dynamics to explicitly and implicitly match prescribed lateral and fore-aft force profiles, respectively. Forward dynamic simulations incorporating the prescribed leg actuation protocol are employed to identify periodic orbits for gaits in which the leg acts to either push the body away from or pull the body towards the foot placement point. Gait stability and robustness to external perturbation are found to change significantly as a function of slope for each type of gait.

INTRODUCTION

Individual leg function in sprawled-posture insects differs dramatically between running on the level and climbing vertically, with legs pushing towards the body centerline on the level [1] and pulling towards the foot placement point when climbing [2]. Exactly when and why this functional change happens remains unexplained, but we hypothesize that improved gait stability and robustness to external perturbations provide impetus for the switch. Such lateral plane instabilities may also have relevance for dynamic walking and running machines that operate on inclines, and the model investigated here may have relevance to the design and development of such robots.

METHODS

This study employs a point mass lateral leg spring model [3] restricted to move in the lateral plane aligned along a slope of angle σ (-90 to 90 degrees). Forces acting on the mass consist of those due to the actuated elastic leg and the component of gravity that acts along the slope.

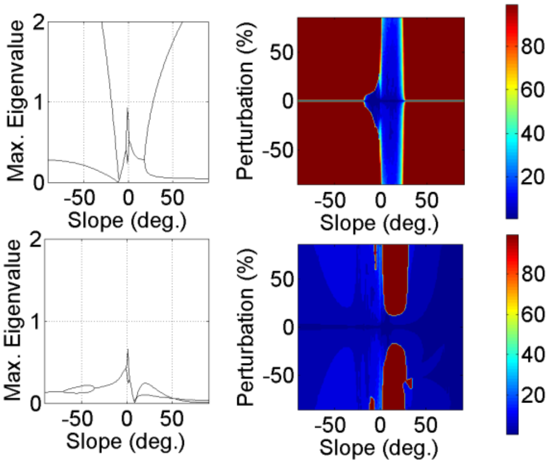

Figure 1: Periodic gait eigenvalues and the number of stance phases required to recover from external perturbations for pushing (top) and pulling (bottom).

Comparing experimental data for cockroaches running on the level [1] and climbing [2] reveals that the average, filtered fore-aft force profile changes from $F_L \sin(2\Omega t)$ $(\sigma = 0)$ to $F_V \sin(\Omega t)$ $(\sigma = 90)$. In the absence of experimental data regarding force profiles for intermediate slopes, we approximate the fore-aft force profile as

$$F_{FA} = A(\sigma) sin(2\Omega t) + B(\sigma) \sin(\Omega t).$$

Ascending or descending a slope in a periodic fashion requires equal fore-aft velocities at the beginning of each stance phase. Such periodic gaits require that the average of the time rate of change of momentum during a stance phase equal zero, yielding $B(\sigma) = mg\pi \sin(\sigma)/2$. The coefficient $A(\sigma)$ is chosen to vary in a similar fashion and to match fore-aft forces for $\sigma = 0$, such that $A(\sigma) = -.196mg[1 - sgn(\sigma) sin(\sigma)]$. Freedom exists in the choice of the lateral force profile; here it is chosen to vary in a manner similar to the fore-aft profile and to match the profiles evidenced experimentally for $\sigma = 0$ and $\sigma = 90$

$$F_{LAT} = \begin{bmatrix} 0.156(1 - sgn(\sigma) \sin(\sigma)) + \\ 0.24\, sgn(\sigma) \sin(\sigma) \end{bmatrix} \text{mg} \sin(\Omega t).$$

Integrating the fore-aft and lateral force profiles yields expressions for the desired center of mass motion (x_m, y_m) throughout a stance phase. The force-free leg length is determined by equating the force developed in the elastic leg to the lateral force profile

$$l(t) = (1 + F_{LAT}) \frac{|q|}{k q_x}$$

where k is the spring constant (4 N/m), $|q| = \sqrt{q_x^2 + q_y^2}$, and q_x and q_y represent the difference between the mass center location and the foot placement.

RESULTS AND DISCUSSION

Prescribed leg length variations generated by inverse dynamics are utilized in forward dynamic simulations of the equations of motion. Periodic gaits are identified at the preferred average forward velocity of the cockroach *Blaberus discoidalis*, 0.225 m/s, over the entire range of slopes. In Fig. 1, eigenvalues of the linearized Jacobian indicate that pushing gaits destabilize below and above negative 19 and positive 27 degrees, respectively, while pulling gaits are stable over the entire range. Gait stability results for this speed, when analyzed in conjunction with recovery rates from external lateral perturbations of up to 85% of the system's linear momentum, suggest that pushing gaits are likely preferable for $\sigma = -10$ to 20 degrees. Similar analyses at higher and lower average forward velocities suggest that this transition point varies with speed.

REFERENCES

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