An Analytical Measure of the Control Authority over the Unactuated Degree of Freedom in Underactuated Bipeds

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Summary

This work examines how an analytical measure of the dynamic coupling between the controlled and uncontrolled velocities in a biped underactuated by one control can be used to design more robust dynamic walking gaits. A simple two-link biped example demonstrates the approach.

Introduction

Recent work has developed a general means to determine the dynamic coupling between the controlled and uncontrolled velocity directions of an underactuated mechanical system and has applied it to calculate the control inputs that will drive a mechanical system underactuated by one control to rest for an arbitrary initial configuration and velocity [1, 2]. Applying this concept to biped locomotion control, the objective is not to drive the system to rest, but rather to design gaits that exploit the coupling/decoupling between the controlled and uncontrolled velocity directions. In decoupled configurations, disturbance rejection is poor because external disturbances in the uncontrolled directions are impossible to reject until the biped moves to a configuration where there is coupling. In such configurations, however, disturbance isolation is strong because disturbances in the direction of the controlled velocities are isolated from the uncontrolled directions. The opposite strengths/weaknesses are found in highly coupled configurations. The analytical measure of coupling can be computed for any biped, but here, an example of the simplest biped illustrates its utility.

Example

The 2×2 inertia tensor $\mathbb{G}$ for a two-link compass walker is

$$\begin{aligned} \mathbb{G}_{1,1} &= J + M(l-l_c)^2 \\ \mathbb{G}_{1,2}, \mathbb{G}_{2,1} &= -J - M(l-l_c)(l-l_c-\cos\theta_1) \\ \mathbb{G}_{2,2} &= 2J + Ml_c^2 \\ &\quad + M(l^2 - 2l(l-l_c)\cos\theta_1 + (l-l_c)^2) \end{aligned}$$

where m, l, and J are the mass, length, and moment of inertia of each leg, l_c is the leg's center of mass location relative to the distal end, θ_1 is the swing leg angle relative to the stance leg, and θ_2 is the stance leg angle relative to the vertical (not found in $\mathbb{G}$ due to the chosen coordinates). The kinetic energy is $T = \frac{1}{2}\dot{\theta}^T \mathbb{G}\dot{\theta}$, where $\dot{\theta} = \left[\dot{\theta}_1\ \dot{\theta}_2\right]^T$. The potential energy is $V = mg((l+l_c)\cos\theta_2 - (l - l_c)\cos(\theta_1 - \theta_2))$. For $L = T - V$, the equations of motion are given by Lagrange's equations $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i} = F_i$, where $F = \tau_1 \left[1\ 0\right]^T$. Considering the velocity component w_1 aligned with the input and the velocity component s orthogonal to the input with respect to the kinetic energy metric, a velocity is expressed $\dot{\theta} = w_1 Y + sY_\perp$, where $Y = \mathbb{G}^{-1}[1\ 0]^T$, $Y_\perp = Y_* / \sqrt{Y_*^T \mathbb{G} Y_*}$, and $Y_* = [0\ 1]^T$.

The time derivative of the uncontrolled velocity s is

$$\begin{aligned} \frac{d}{dt}s(t) = &-w_1^2(t)\left(\frac{\partial Y^k}{\partial \theta^i}Y^i + \Gamma^k_{ij}Y^iY^j\right)\mathbb{G}_{kl}Y^l_\perp \\ &-s(t)w_1(t)\left(\frac{\partial Y^k_\perp}{\partial \theta^i}Y^i + \Gamma^k_{ij}Y^iY^j_\perp\right)\mathbb{G}_{kl}Y^l_\perp \\ &-s^2(t)\left(\frac{\partial Y^k_\perp}{\partial \theta^i}Y^i_\perp + \Gamma^k_{ij}Y^i_\perp Y^j_\perp\right)\mathbb{G}_{kl}Y^l_\perp - \frac{\partial V}{\partial \theta^l}Y^l_\perp, \end{aligned}$$

where Γ are the Christoffel symbols as usually defined. The coefficient of the first term quantifies the coupling associated only with the controlled velocity w_1 and is zero for this example. The coefficient $\left(\frac{\partial Y^k_\perp}{\partial \theta^i}Y^i + \Gamma^k_{ij}Y^iY^j_\perp\right)\mathbb{G}_{kl}Y^l_\perp$ in the second term quantifies the coupling associated with both the controlled and uncontrolled velocities, and being non-zero in this example, it is the analytical measure of the control authority over the unactuated degree of freedom. The controlled velocity does not appear in the fourth or fifth term, so while they affect the change in the uncontrolled velocity, they provide no measure of control authority.

Results

Simulation results with the two-link biped demonstrate that using a feedback controller based on the hybrid zero dynamics approach [3] to induce dynamic walking, failure to complete a step is predicted by a reduction in the control authority below a minimum value at critical junctures in a step. This is observed both in altering the initial conditions for a step and simulating uneven terrain with small, random changes in ground height for touchdown. The control authority information, combined with an understanding of the nature of typical disturbances, can be used to design more robust dynamic walking gaits in underactuated bipeds.

References

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[3] E.R. Westervelt, J. Grizzle, & D. Koditschek. *IEEE Trans. Auto. Control*, 48(1):42-56, 2003.