Feedback Controller Parameter Optimization for Standing Balance

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We are exploring a hypothesis of Park, Horak, and Kuo [1] that postural feedback gains in standing balance should change with perturbation size. From an engineering point of view this is known as gain scheduling. We use an optimization approach to see if feedback gains scale with the perturbation for a simulated robot. We simulate the robot as a two joint inverted pendulum, use a horizontal push of a given size and location as a perturbation at any given time, and optimize parametric controllers for different push sizes and locations. During a simulated perturbation experiment, the appropriate controller is continuously selected based on the current push. For a constant push, the simulated robot moves to an equilibrium position which leans into the push and has zero ankle and hip torques. We compare the performance of optimized controllers with parameters that are linear, quadratic, and cubic in the error state.

This study uses a two-link inverted pendulum model, with actuators located at the ankle and hip joints. We define the state as ankle and hip angles and angular velocities, and the cost function as

$$Cost = \sum \left(\Delta \mathbf{x}^T \mathbf{Q} \Delta \mathbf{x} + \boldsymbol{\tau}^T \mathbf{R} \boldsymbol{\tau} \right),$$

where $\Delta\mathbf{x}$ is the state error, $\boldsymbol{\tau}$ represents torque, and $\mathbf{Q}$ and $\mathbf{R}$ are the weight matrices.

We consider three types of feedback controller. The first is a linear controller which acts on the error in each state variable

$$\tau_a = k_1 \Delta\theta_a + k_2 \Delta\theta_h + k_3 \Delta\dot{\theta}_a + k_4 \Delta\dot{\theta}_h,$$
$$\tau_h = k_5 \Delta\theta_a + k_6 \Delta\theta_h + k_7 \Delta\dot{\theta}_a + k_8 \Delta\dot{\theta}_h,$$

where

$$\Delta\theta_a = \theta_a - \theta_{a\_d},\ \Delta\theta_h = \theta_h - \theta_{h\_d},$$
$$\Delta\dot{\theta}_a = \dot{\theta}_a - \dot{\theta}_{a\_d},\ \Delta\dot{\theta}_h = \dot{\theta}_h - \dot{\theta}_{h\_d},$$

represent the error between actual state and the equilibrium state for the current push.

The second controller is a quadratic controller which has as terms all linear and quadratic terms constructed from the elements of the error vector:

$$\tau_a = k_1 \Delta\theta_a^2 + k_2 \Delta\theta_a \Delta\theta_h + \ldots,$$
$$\tau_h = k_{16} \Delta\theta_a^2 + k_{17} \Delta\theta_a \Delta\theta_h + \ldots.$$

The third controller is a cubic controller which has as terms all linear, quadratic, and cubic terms constructed from the elements of the error vector:

$$\tau_a = k_1 \Delta\theta_a^3 + k_2 \Delta\theta_a^2 \Delta\theta_h + \ldots,$$
$$\tau_h = k_{36} \Delta\theta_a^3 + k_{37} \Delta\theta_a^2 \Delta\theta_h + \ldots.$$

The torques output from the controllers are limited to the following range $|\tau_a| \le 50N$ and $|\tau_h| \le 157N$.

We optimize parameter sets for each of these controllers at a number of push sizes and locations. We assume all pushes are horizontal, as vertical pushes have little effect. For each push size and location we first calculate a desired state, which is not standing straight up but is the posture where the robot leans into the push and the torques at the ankle and hip are zero. Starting with the robot standing vertically as the initial state, we can compute optimized parameters of each controller for each possible push case, by using the Nelder and Mead method. Supposing the push changes between -38N and 38N with a step size of 1N, at four different locations, we have 304 controllers in total. The ankle and hip trajectories of Figure 1 are achieved by applying linear controllers for pushes with variable size and location. Figure 2 shows big changes of the hip gains for a linear controller for different push size while less than 8% changes of the ankle gains.

The results above will be tested on our robot platform. We will also study pushes during walking.

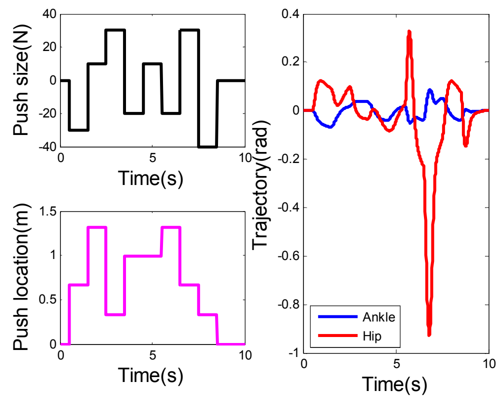

Figure 1: The trajectories generated by the parameterized linear controller for pushes with variable size and location.

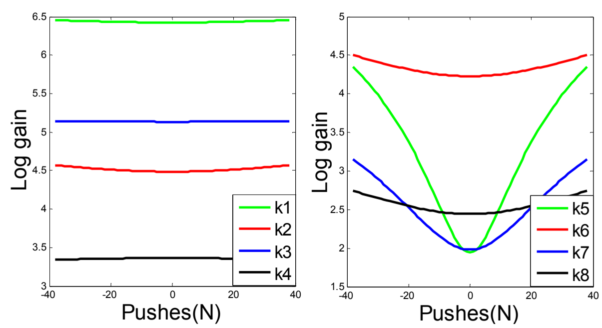

Figure 2: The gain of the linear controller using linear quadratic regulation designed at the equilibrium state of pushes located at the head of the robot.

References

1. Park, S., Horak, F. B., and Kuo, A. D. Postural feedback responses scale with biomechanical constraints in human standing. *Exp. Brain Res.*, **154**, 417-427, 2004