Mathematical Tools for Tracking Uncertainty Through Gait

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SUMMARY

Tracking uncertainty in walking is critical for understanding how the body responds to the innate lack of precision in the measurement of biological systems. By utilizing different mathematical techniques, we can track this uncertainty through gait and explore how control could compensate for these inexact measures.

INTRODUCTION

To understand the nature of gait stability, it is important to explore how the body adapts to uncertainty, with and without control. For this reason, we must have objective mathematical tools that can track this uncertainty. Some simple techniques that can be used to follow this uncertainty as it propagates through the gait cycle are linearizing each phase of the system, linearizing impact restitution laws, and using Monte Carlo methods.

METHODS

Consider a system modeling gait dynamics

$$\begin{aligned}\dot{x} &= f(x,u)\\ u(t) &= \mu(t) + k(t)(x_{ref} - x)\end{aligned}$$

where x is the state space, μ the feedforward control, and u the total control.

One method is to linearize the system around a "stable" reference trajectory x_{ref}. This trajectory can be calculated using mathematical techniques such as projection operators or taken from experimental data. Linearizing the system allows us to track the uncertainty z through the phase as follows

$$\dot{z} = A(t)z + B(t)u$$

where $A(t)$ is the Jacobian with respect to x evaluated with the feedforward state space variables and $B(t)$ is the Jacobian with respect to the control. A similar method can be used for phase changes, modeled using impact restitution laws. By linearizing around this map, the uncertainty at the end of one phase can be translated to the beginning of the next phase.

Another method is Monte Carlo simulation, which samples initial conditions around a measured value and runs the model dynamics. In this way, we can test how variance is translated through the phase. Though considered a "brute force" method, it is useful for exploratory research if used systematically.

RESULTS & DISCUSSION

When linearizing non-linear systems, like the stance phase of an actuated monopod [3] with LQR control, there is minimal variation between the uncertainty predicted by the linearization and the actual difference between the desired and measured trajectories (Figure 1). Though this figure depicts the approximation for the vertical position of the CoM, other state-space variables exhibited similar patterns. This method can also be used for trajectories with noise or discontinuities, including experimental data, or, as mentioned previously, for modeling phase transition. However, error can be introduced when going from the continuous phase to the discrete system necessary for modeling impact.

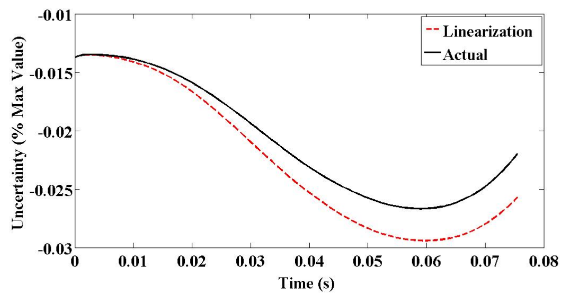

FIGURE 1

Including a uniform uncertainty of 10% of the maximum of each state space variable and running a Monte Carlo simulation of the swing phase of the monopod leads to an assymetric distribution (Figure 2) of sampled points (black diamonds) around the measured value (red circle) due to the control's limiting of the leg swing velocity. This method, though it can track the non-linearities in the system, is computationally expensive.

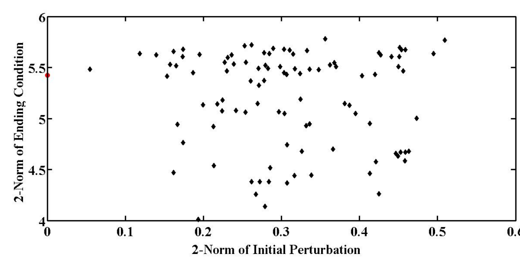

FIGURE 2

Lastly, both techniques assume the stable trajectory is the desired trajectory, whereas people are likely not optimizing to a specific trajectory, but any trajectory within a stable annulus in state space.

REFERENCE

M. Ahmadi and M. Buehler.*IEEE T. Robotic Autom.*, **13** (1):96-104, 1997.